Hall-Lorenz ratio of $YBa_2Cu_3O_7$ using Ionization energy based Fermi-Dirac statistics and charge-spin separation

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The temperature dependent properties of heat capacity, heat conductivity and Hall-Lorenz ratio have been solved numerically after taking the previously proposed ionization energy based Fermi-Dirac statistics and the coexistence of Fermi and charge-spin separated liquid into account. The thermo-magneto-electronic properties are entirely for spin and charge carriers, hence the phonon contribution has been neglected. A linear dependence between the Hall-Lorenz ratio and temperature (T) is also obtained in accordance with the experimental results for overdoped $YBa_2Cu_3O_{7-\delta}$, if these conditions, $E_I < T_c$ with respect to resistivity and there are no spinon pairings ($T^* = 0$) are satisfied. Heat conductivity based on both pure and electron-contaminated charge-spin separated liquid in *ab*-planes above T_c are found to increase with decreasing T as a consequence of inverse proportionality with T. The T-dependence of heat capacity are also highlighted, which qualitatively complies with the experimental findings.

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1. INTRODUCTION

The mechanism of superconductivity in $YBa_2Cu_3O_{7-\delta}$ has got to be of both electronic and magnetic in nature. However, its transport properties in the normal state (above T_c) of high-T_c superconductors (HTSC) are still clouded with obscurities due to anisotropy and strong correlations. On the one hand, there is a lack of confluence on the interpretations of ARPES (Angle-Resolved Photoemission Spectroscopy) and scanning tunnelling spectroscopic (STS) measurements convincingly with respect to superconductor, spin and charge gaps and its evolutions, as well as the holon condensation and precursor superconductivity (Cooper pairs above T_c) [1, 2, 3, 4]. On the other hand, the charge and spin carriers in the normal state seem to obey both Fermi and non-Fermi liquid [5, 6, 7, 8] somewhat mimicking the duality property of light (particle-wave). Instead of assuming that there are no such connections of which the normal state properties are somehow influenced by the types of experiments, one can entirely and safely neglect this conception by heavily relying on the basic transport experiments such as the resistivity, Hall resistance, Hall-Lorenz ratio, heat capacity and conductivity. Anyhow, the normal state properties with characteristics or spin gap temperature (T^*) and $T_{crossover}$ must consist of at least two types of strongly interacting liquid. Theoretical studies based on other experimental techniques, such as Optical, STS, ARPES and NMR (Nuclear Magnetic Resonance) are also essential and need to be carried out in order to extract the missing puzzles, which could act as a bridge between the theoretical interpretations based on transport experiments and other experimental techniques.

In addition, the former focus-point based on basic transport experiments also suggests that not all magneto-electronic properties and its magnitudes determined from the STS, optical, NMR and ARPES measurements are directly applicable for the basic transport mechanisms and vice versa. This scenario arises as a result of the principle of least action that always plays a crucial and dominant role in any transport measurements [9], which is actually reinforced by the level of anisotropy. As an example, the magnitude of energy gaps for ferromagnetic manganites above Curie temperature (paramagnetic phase), doped semiconductors with limited solubility of the dopants and even in semiconducting normal states of superconductors determined from the resistivity measurements are always smaller than the ones measured and/or calculated from other experimental techniques, including STS. Consequently, the intense focus here will be given on the basic transport experiments in order to explain the thermo-magneto-electronic properties of $YBa_2Cu_3O_{7-\delta}$ theoretically. It is well known that the popular theoretical proposals like, stripes, pure charge-spin separation (CSS), polaron-bipolaron model, quantum critical points and marginal Fermi liquid [5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17] seriously lack the ability in one way or another to predict all the transport properties consistently and convincingly in the presence of electric field, magnetic field and heat gradient or any of its combinations in the temperature range between T_c and 300 K. Instead of considering the pure CSS liquid, CSS in the midst of electrons is used here to comply, first with all the basic transport measurements consistently before embarking on other experimental results. The reason for choosing CSS related mechanism is that it works extremely well qualitatively for the resistivity and Hall coefficient-angle measurements at T ranging from 300 K $\rightarrow T_c$, taking

care of both T^* and $T_{crossover}$ effects.

In the early developments for the theory of HTSC, Lee and Nagaosa [18] have predicted the violation of Wiedemann-Franz law ($\kappa\rho/T$; κ and ρ denote the heat conductivity and electrical resistivity respectively) theoretically based on pure CSS. This breakdown was thought to be solely as a result of incompatibility between the heat and charge conductivity rules, which turns out to be insufficient and additional constraints are required [19]. The charge conductivity obeys the parallel-circuit's resistance rule, presently known as the Ioffe-Larkin rule [20] whereas the heat conductivity follows the series-circuit's resistance rule [18]. Unfortunately, CSS has been found to be energetically unfavorable in 2-dimensional (2D) systems, unlike in 1D systems as pointed out by Sarker [21]. Sarker showed that pure CSS liquid gives rise to excessive kinetic energy relative to lowest possible kinetic energy, which will eventually prohibit stable recombination processes for the occurrences of superconductivity in 2D and Fermi liquid in 3D. Nevertheless, CSS that leads to spinons and holons as spin and charge carriers respectively were allowed to coexist with electrons in ab-planes recently so as to fulfill the dimensionality crossover [22, 23, 24] as well as to avoid this excessive kinetic energy scenario. In Ref. [24], note that there are three typographical errors in this paper, two of them can be found in Eq. (9) where $(m_e^* m_h^*)^{-3/4}$ and $(2\pi\hbar^2/k_B)^{-3/2}$ should be replaced with $(m_e^* m_h^*)^{-1/4}$ and $(2\pi\hbar^2/k_B)^{3/2}$ respectively. Therefore, the fitting parameter, A_2 is actually equals to $(A_2/2e^2)(m_e^* m_h^*)^{-1/4}(2\pi\hbar^2/k_B)^{3/2}$. The electrons here obey the ionization energy based Fermi-Dirac statistics [25, 26] (iFDS). A hybrid model based on this coexistence of Fermi and charge-spin separated (FCSS) liquid was proposed by employing the Ichinose-Matsui-Onoda (IMO) equation [27] that has been successful to explain both c-axis and ab-plane's resistivities ($\rho_c(T)$, $\rho_{ab}(T)$) as well as its dimensionality crossover including the Hall resistances [19] ($R_H^{(c)}$, $R_H^{(ab)}$). In the presence of magnetic field (**H**), FCCS liquid also obeys the Anderson's hypothesis [28], $1/\tau_{transport} \propto T$ and $1/\tau_{Hall} \propto T/\tau_{transport} \propto T^2$. Likewise, it is stressed here that the thermo-magneto-electronic properties of $YBa_2Cu_3O_{7-\delta}$ can also be explained using the FCSS liquid without any additional assumptions or constraints, apart from the ones given in Ref. [19]. However, note that the thermal related physical parameters derived here neglect the phonon contribution.

2. THEORETICAL DETAILS

2.1. Heat capacity and related distribution functions

Holon (spinless charged-boson), spinon (chargeless fermion) and electron can carry heat (entropy). The respective particles can be excited to a higher energy levels satisfying the Bose-Einstein statistics ($f_{BES}(E)$), Fermi-Dirac statistics ($f_{FDS}(E)$) and ionization energy based Fermi-Dirac statistics ($f_{iFDS}(E)$). Therefore, the heat capacity in ab-planes can be explicitly written as

$$\begin{aligned}\mathcal{C}_{ab} &= DOS\Bigg[\int_0^\infty (E-E_F)\frac{\partial f_{BES}(E)}{\partial T}dE \\ &\quad + \int_{\Delta_{SG}}^\infty (E-E_F)\frac{\partial f_{FDS}(E)}{\partial T}dE \\ &\quad + \int_0^\infty (E-E_F-E_I)\frac{\partial f_{iFDS}(E)}{\partial T}dE\Bigg] \\ &= \frac{1}{\pi\hbar^2}\Bigg[m_s\Phi_{FDS}(T) \\ &\quad + m_h\Phi_{BES}(T) + m_e^*\Phi_{iFDS}(T)\Bigg]. \qquad (1)\end{aligned}$$

The E-independent 2D density of states is, $DOS = m^*/\pi\hbar^2$. $\hbar = h/2\pi$, h denotes Planck constant, k_B is the Boltzmann constant while m^* represents the effective mass. $\Phi_{BES}(T)$, $\Phi_{FDS}(T)$ and $\Phi_{iFDS}(T)$ denote the integrals with respect to BES, FDS and iFDS respectively. In the case of pure CSS liquid, the term, $m_e^*\Phi_{iFDS}(T) \rightarrow 0$. The respective distribution functions for BES, FDS and iFDS are given by $f_{BES}(E) = 1/[\exp\left[(E-E_F)/k_BT\right]-1]$, $f_{FDS}(E) = 1/[\exp\left[(E-E_F)/k_BT\right]+1]$ and [22, 23, 26] $f_{iFDS}(E) = 1/[\exp\left[(E-E_F+E_I)/k_BT\right]+1]$. E_I is the ionization energy, $E_I = e^2/8\pi\epsilon\epsilon_0 r_B$ and $\Delta_{SG} = T^*$, denotes the spin gap in ab-planes only. ϵ and ϵ_0 are the dielectric constant and permittivity of free space respectively, whereas e and r_B represent the charge of an electron and the Bohr radius respectively. Importantly, the variation of E_I with **H** indicates that r_B varies accordingly with **H**. Identical relationship was also given between polaronic radius, r_p and polaronic hopping energy, E_p by Banerjee *et al.* [29]. The crucial issue here is that E_I implicitly represent the polaronic effect. Now switching back to the heat capacity, note that the relation, $\mathcal{C}_{ab} = \mathcal{C}^s + \mathcal{C}^h + \mathcal{C}^e$ directly comes from $\kappa = \kappa_s + \kappa_h + \kappa_e$ that follows from the total heat current in $\kappa_{ab} = -\sum_\alpha j_Q^\alpha \nabla T = \sum_\alpha \mathcal{C}^\alpha v_F^2 \tau_\nu/2$, α = spinon (s), holon (h), electron (e) and ν = transport ($ab(s,h)$), electron (e).

2.2. Heat conductivity and Hall-Lorenz ratio

The 2D non-phononic heat conductivity in ab-planes, $\kappa_{ab}(T)$ is given below after taking $E_F = \frac{1}{2}m^* v_F^2$, $\tau_{ab(s,h)}$ (spinons and holons interactions in ab-planes, neglecting spinon-pairing) and τ_e (electron-electron interactions) into account.

$$\kappa_{ab} = \kappa_s + \kappa_h + \kappa_{s+h\rightarrow e} = \kappa_s + \kappa_h + \gamma\kappa_e$$

$$= \frac{E_F}{\pi\hbar^2}\Bigg[\tau_{ab(h)}\Phi_{BES}(T) + \tau_{ab(s)}\Phi_{FDS}\big(T, \lim_{\Delta_{SG}\to 0}\big) + \gamma\tau_e\Phi_{iFDS}(T)\Bigg]$$
$$= \frac{E_F}{\pi\hbar^2}\left[\frac{\Phi_{BES}(T)}{B_hT} + \frac{\Phi_{FDS}(T)}{B_sT^{4/3}} + \gamma\frac{\Phi_{iFDS}(T)}{A_eT^2}\right]. \quad (2)$$

The ab-plane scattering rate for the spinons and holons are respectively given by [18, 27] $\tau_{ab(s)} = 1/B_sT^{4/3}$ and $\tau_{ab(h)} = 1/B_hT$. E_F and v_F denote Fermi energy and Fermi velocity respectively whereas γ is the constant of proportionality [19, 24] that represents the contribution of electrons in ab-planes. m_s, m_h and m_e^* are the holon, spinon and electron's effective mass respectively. B_s, B_h and A_e are the spinon, holon and electron's scattering rate dependent constants respectively (independent of T) in the presence of ∇T. Unlike in the previous electrical resistivity derivations [19, 27], spinon pairing has been neglected in this subsequent heat conductivity calculations since τ_{ab} in the presence of spinon-pairing and ∇T is still unclear. In other words, the IMO's $\tau_{ab(h)}$, $1/\tau_{ab(h)} = BT[1-C(T^*-T)^d]$ in the midst of spinon-pairing may not represent the above $\tau_{ab(h)}$ correctly. As such, the integral, $\Phi_{FDS}(T)$ now reads as $\Phi_{FDS}(T, \lim_{\Delta_{SG}\to 0}) = \int_0^\infty (E-E_F)\frac{\partial f_{FDS}(E)}{\partial T}dE$. Note that all those integrals were solved numerically. It is also worth mentioning that even if one assumes, $\exp[(E-E_F)/k_BT] \gg 1$ and $\exp[(E-E_F+E_I)/k_BT] \gg 1$, the results remain consistent with the numerical ones (without such assumptions) as anticipated. Subsequently, the heat conductivity for the pure CSS liquid can be written as

$$\kappa_{ab} = \kappa_h + \kappa_s = \frac{E_F}{\pi\hbar^2}\left[\frac{\Phi_{BES}(T)}{B_hT} + \frac{\Phi_{FDS}(T, \lim_{\Delta_{SG}\to 0})}{B_sT^{4/3}}\right]. \quad (3)$$

On the other hand, the c-axis heat conductivity, $\kappa_c(T)$ is simply given by

$$\kappa_c = \kappa_{e\rightleftharpoons s+h} + \kappa_e = \beta(\kappa_h + \kappa_s) + \kappa_e$$
$$= \frac{E_F}{\pi\hbar^2}\Bigg[\beta\tau_{ab(h)}\Phi_{BES}(T) + \beta\tau_{ab(s)}\Phi_{FDS}\big(T, \lim_{\Delta_{SG}\to 0}\big) + \tau_e\Phi_{iFDS}(T)\Bigg]$$
$$= \frac{E_F}{\pi\hbar^2}\left[\beta\frac{\Phi_{BES}(T)}{B_hT} + \beta\frac{\Phi_{FDS}(T)}{B_sT^{4/3}} + \frac{\Phi_{iFDS}(T)}{A_eT^2}\right]. \quad (4)$$

Here, β is the constant of proportionality [19, 24] that represents the contribution of spinon-holon in c-axis. As a matter fact, it is stressed that the origin of γ and β in Eqs. (2) and (4) are as a result of the following definitions, (which is in compliance with the mechanism for the resistivity [19]) the term $\kappa_{s+h\to e}$ is defined to be the heat conductivity reduced by the process $s+h \to e$ occurring in the ab-planes. Alternatively, it is the heat conductivity reduced by the electrons in ab-planes. If $s+h \to e$ is completely blocked in ab-planes then $\kappa_{s+h\to e}$'s contribution is zilch. Any increment in κ_e also increases $\kappa_{s+h\to e}$ therefore $\kappa_{s+h\to e} = \gamma\kappa_e$. In contrast, the term $\kappa_{e\rightleftharpoons s+h}$ is defined to be the heat conductivity reduced by the blockage in the process, $e \rightleftharpoons s+h$ or the reduction in heat conductivity caused by the blockage faced by electrons to enter the ab-planes ($e \to s+h$) and the blockage faced by spinons and holons to leave the ab-planes ($s+h \to e$). These blockages originate from the non-spontaneity conversion of $e \rightleftharpoons s+h$. The reduction in heat conductivity, $\kappa_{e\rightleftharpoons s+h}$ can also be solely due to the blockage of $e \to s+h$ or $s+h \to e$. Actually, if the magnitude of blockage in $e \to s+h > s+h \to e$ then the blockage of $e \to s+h$ contributes to κ_c. In short, if one of the conversion, say $e \to s+h$ is less spontaneous than $s+h \to e$, then the former conversion determines the $\kappa_{e\rightleftharpoons s+h}$. Moreover, reduction in κ_{ab} further blocks $e \rightleftharpoons s+h$ that leads to a reduction in $\kappa_{e\rightleftharpoons s+h}$ hence, $\kappa_{e\rightleftharpoons s+h} = \beta\kappa_{ab}$. I.e, the process $e \rightleftharpoons s+h$ becomes increasingly difficult with reduction in κ_{ab}. This proportionality can also be interpreted as the additional scattering for the electrons to transfer heat directionally across ab-planes. If $e \rightleftharpoons s+h$ is spontaneous then the term, $\kappa_{e\rightleftharpoons s+h}$ is null. Subsequently, the Hall-Lorenz number is defined as $L_H^{(ab)} = \rho_{ab}^H(T)k_{ab}(T)/T$. As such, $L_H^{(ab)}$ can be expressed as

$$L_H^{(ab)} = \frac{1}{T}\left[\frac{m_h}{n_he^2\tau_{tr}} < T > + \frac{\gamma m_e^*}{n_ee^2\tau_e}\right] \times \frac{E_F}{\pi\hbar^2}\Bigg[\tau_{ab(h)}\Phi_{BES}(T) + \tau_{ab(s)}\Phi_{FDS}(T, \lim_{\Delta_{SG}\to 0}) + \gamma\tau_e\Phi_{iFDS}(T)\Bigg]$$
$$= \left[B_{LH}\frac{m_h}{e^2n_h}T + \gamma A_{LH}\frac{\pi\hbar^2}{k_Be^2}\exp\left(\frac{\Delta_{PG}}{T}\right)\right] \times \frac{E_F}{\pi\hbar^2}\left[\frac{\Phi_{BES}(T)}{B_hT} + \frac{\Phi_{FDS}(T, \lim_{\Delta_{SG}\to 0})}{B_sT^{4/3}} + \gamma\frac{\Phi_{iFDS}(T)}{A_eT^2}\right]. \quad (5)$$

Notice that the resistivity term in Eq. (5) originates from the FCSS liquid, $\rho_{ab} = \sigma_s^{-1} + \sigma_h^{-1} + \sigma_{s+h\to e}^{-1} = \sigma_s^{-1} + \sigma_h^{-1} + \gamma\sigma_e^{-1} = \rho_{ab} + \gamma\rho_c$ that have been derived in Ref. [19, 24]. the term $\sigma_{s+h\to e}^{-1}$ is defined to be the resistivity caused by the process $s+h \to e$ occurring in the ab-planes. Alternatively, it is the resistivity caused by the electrons in ab-planes. If $s+h \to e$ is completely blocked in ab-planes then $\sigma_{s+h\to e}^{-1} = 0$. Any increment in σ_e^{-1} also increases $\sigma_{s+h\to e}^{-1}$ therefore $\sigma_{s+h\to e}^{-1} = \gamma\sigma_e^{-1}$. In addition, the stated resistivity also neglects the spinon-pairing contribution, as required by the heat conductivity equation derived earlier

in the absence of spinon-pairing effect. B_{LH} and A_{LH} are holon and electron's scattering rate dependent constants (independent of T) in the vicinity of both $\mathbf{H}$ and ∇T. The additional T contribution is noted with $< T >$ and [25] $n_e \approx \sqrt{np} = \frac{k_B T}{\pi\hbar^2}(m_e^* m_h^*)^{1/2}\exp(\frac{-E_I}{k_B T})$ in which, n_h and n_e are the holon and electron's concentration respectively. Note that the ionization energy (E_I) denotes the electron's charge Pseudogap (Δ_{PG}). In accordance with Refs. [19, 28], there are two types of scattering rates with respect to T-dependence namely, $\tau_{tr} \propto 1/T$ and $\tau_H \propto \tau_{tr}/T$. Large phase space of $1/T$ is required for τ_H due to spinon-holon pair scattering in the presence of $\mathbf{H}$ since both spinon and holon are at the mercy of $\mathbf{H}\times\mathbf{E}$ unlike in the presence of electric field ($\mathbf{E}$) only. Now, considering the pure CSS liquid, one can show that Eq. (5) can be reduced to

$$L_H^{(ab)} = \frac{m_h B_{LH} E_F}{e^2 n_h \pi\hbar^2}\left[\frac{\Phi_{BES}(T)}{B_h} + \frac{\Phi_{FDS}(T)}{B_s T^{1/3}}\right]. \quad (6)$$

At $T_{crossover} > T_c$, the linearity of $L_H^{(ab)}(T)$ for pure CSS liquid will be inadequate as a result of $T_{crossover}$ (Δ_{PG}) effect from $\rho_{ab}(T)$. Recall that both $\kappa_{ab}(T)$ and $L_H^{(ab)}(T)$ are unsuitable below T^* as explained earlier. Anyway, all the derivations thus far contain no uncontrolled approximations or any additional assumptions, apart from the ones stated here and in Ref. [19].

3. ANALYSIS

The calculated heat capacity in ab-planes, $\mathcal{C}_{ab}(T)$ is found to decrease linearly with T above T_c (normal state). In the case of $E_F > T_c$, there is a crossover in $\mathcal{C}_{ab}(T)$ and note that it does not represent the heat capacity jump due to Cooper pairs and/or condensed Bosons because this crossover is an exponential one without a drastic jump. Figure 1a)-d) depict the variation of $\mathcal{C}_{ab}$ with T for both FCSS and CSS liquid. It is worth noting that both $T_{crossover}$ and T^* tend to deviate $\mathcal{C}_{ab}(T)$ downward from T-linear opposing the exponential increase due to E_F. This is because of the proportionality, $\mathcal{C}_{ab}(T) \propto \exp[E_F/k_B T] + \exp[(E_F - E_I)/k_B T] + \exp[(E_F - \Delta_{SG})/k_B T]$. Interestingly, the indirectly measured electronic heat capacity by Loram *et al.* [32] at $T > T_c$ (with T-linear property and a slight upward curve near T_c, see Fig. 10 in Ref. [32]) is identical with the calculated $\mathcal{C}_{ab}(T)$. The parameters used to obtain those plots are also given in the figure itself. Of course, E_I is absent in Fig. 1c)-d) since the latter two plots are for pure CSS liquid. Interestingly, those results were obtained regardless of the statistics used, FDS or BES since both Fermions and Bosons carry entropy identically at higher temperatures, above E_F. Simply put, large E_F suppresses the ability of both Fermions and Bosons to transfer heat and instead gives rise to the exponentially-increasing heat capacity below E_F. Notice that all energies were given in Kelvin (K).

The calculated plots for the heat conductivity, $\kappa_{ab,c}(T)$ are given in Fig. 2a)-d). Those curves remained inversely proportional to T, regardless of the reasonable magnitudes of E_F (for CSS curve only) or both, E_F and E_I (for FCSS curve only). Mathematically, the parameter, E_F will give rise to exponential increase of $\kappa_{ab,c}(T)$ if $E_F > T_c$ whereas, E_I leads to an opposite effect in which E_I will slightly reduce the rate of which $\kappa_{ab,c}(T)$ increases with reduction in T if $E_I > T_c$. In other words, the above-stated E_I-effect will only come into play for certain underdoped $YBa_2Cu_3O_{7-\delta}$ compounds that strictly satisfy the condition Δ_{PG} ($T_{crossover}$) $> T_c$ (observable from the resistivity versus temperature measurements). Unlike resistivity, there will not be any observable crossover for $\kappa_{ab,c}(T)$ since only the magnitude of inverse proportionality with T will be reduced, without changing the overall T-dependence. As a consequence, $\kappa_{ab,c}(T)$ will remain inversely proportional to T and the magnitude of this inverse proportionality with T in $\kappa_{ab,c}(T)$ will be reduced (reduced heat conductivity) in certain underdoped $YBa_2Cu_3O_{7-\delta}$ compounds. Similar to $\mathcal{C}_{ab}(T)$, $\kappa_{ab,c}(T) \propto \exp[E_F/k_B T] + \exp[(E_F - E_I)/k_B T]$. Recall that spinon pairings have been omitted ($T^* = \Delta_{SG} = 0$) for $\kappa_{ab,c}(T)$.

The Hall-Lorenz ratio has been the most difficult parameter to be explained by any theory without violating the T-dependence of resistivity, Hall resistance and Hall angle or at least any one of them. Remarkably, the Hall-Lorenz ratio based on FCSS liquid (Eqs. (5) and (6)) after incorporating the $\mathcal{C}_{ab}(T)$, $\kappa_{ab,c}(T)$ and $\rho_{ab}(T)$ reproduces the experimental T-linear property for overdoped $YBa_2Cu_3O_{7-\delta}$, as long as $T_{crossover}$ (E_I from the resistivity) $< T_c$. Note that the E_I from the heat conductivity will deviate $L_H^{(ab)}$ downward while the E_I that originates from the resistivity will give an upward exponential deviation. Such behavior can be trivially verified from these equations, Eqs. (2), (4) and (5). Actually, $T_{crossover}$ is a characteristic of certain underdoped $YBa_2Cu_3O_{7-\delta}$ compounds. The calculated T-linear effect is in excellent agreement with the phonon-independent experimental Hall-Lorenz ratio results [31].

It has been repeatedly mentioned that phonons have been neglected in this work in which, only the electronic version is emphasized rather than the overall thermal effects. The reason being, to investigate the T-dependence of an entirely electronic Hall-Lorenz ratio. The Hall-Lorenz ratio that has incorporated the heat conductivity with phonon contribution also provide the T-linear effect semi-theoretically [19]. It is well established that the resistivity measurements thus far do not require phonon inclusion, which can be understood by realizing that the electron-phonon (*e-ph*) coupling observed via ARPES technique by Lanzara *et al.* [33] actually supports the notion of polaronic effect above T_c in cuprates. One should

note that *e-ph* coupling does not mean that there is a *e-ph* scattering since normal state $\rho(T)$ measurements thus far failed to reveal any *e-ph* scattering (strong T-dependence). Actually, this is not because of $\rho(T)$'s blindness, but due to polarons represented by ionization energy, which gives rise to effective mass of electrons instead of strong T-dependence. The heavier m^* implies the existence of polarons in the normal state of HTSC that also suppresses *e-ph* scattering but not *e-ph* coupling in term of polaronic effect. Similarly, isotope effect (^{18}O, ^{16}O) in cuprates [34, 35, 36] also reinforces the polaronic contribution via *e-ph* coupling rather than *e-ph* scattering. Furthermore, Bloch-Grüneisen formula has been utilized lately [19] to justify convincingly that *e-ph* scattering is not applicable for the resistivity and Hall-Lorenz ratio of $YBa_2Cu_3O_{7-\delta}$.

4. CONCLUSIONS

In conclusion, the temperature dependencies of thermo-magneto-electronic properties such as heat capacity, heat conductivity and Hall-Lorenz ratio, for $YBa_2Cu_3O_{7-\delta}$ has been derived using both electron contaminated CSS and pure CSS liquid. The theoretical plots discussed based on this liquid also shown to be applicable in the experimental thermo-magneto-electronic transport properties, apart from the magneto-electronic transport properties, which have been published previously. Phonon contribution has been neglected throughout the derivations while spinon-pairing is dropped for the derivation of heat conductivity and Hall-Lorenz ratio parameters. The latter is unavoidable since the influence of ∇T on paired spinons and eventually on τ_{ab} is unclear. The assumptions throughout these derivations are again consistently based on Anderson's hypothesis ($\tau_{tr} \propto 1/T$ and $\tau_H \propto \tau_{tr}/T$), the Ioffe-Larkin rule ($\sigma^{-1} = \sigma_s^{-1} + \sigma_h^{-1}$) and FCSS liquid ($\sigma^{-1} = \sigma_s^{-1} + \sigma_h^{-1} + \sigma_e^{-1}$).

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FIG. 1: Theoretical plots for the heat capacity in *ab*-planes, $\mathcal{C}_{ab}(T)$ above 90 K are calculated using Eq. (1). The calculated plots are obtained at different E_I, T^* and E_F. The plots in a) and b) are for FCSS liquid while c) and d) are for pure CSS liquid ($m_e^*\Phi_{iFDS}(T) \to 0$). In a) and b), $\mathcal{C}_{ab}$ with T linear property dominates as long as $E_F < T_c$ in which, an exponential increase takes over below E_F if $E_F > T_c$ and/or $E_F > E_I, T^*$. This latter scenario is due to the fact that both T^* and E_I tend to recover the T-linear effect, opposing E_F. The plots in c) and d) indicate the similarities between CSS and FCCS liquid since E_F is the only parameter in $\mathcal{C}_{ab}(T)$ that is responsible for the large exponential deviation from the T-linear effect. Actually, both T^* and E_I will give rise to a slight downward deviation from the T-linear property.

FIG. 2: The phonon independent heat conductivity in *ab*-planes, $\kappa_{ab}(T)$ of both FCSS and CSS liquid have been plotted in a), b) and c), d) respectively. As anticipated, the variation of $\kappa_{ab}(T)$ with respect to T is weakly inversely proportional to T for all reasonable E_Fs and E_Is. Consequently, $\kappa_{ab}(T)$ from both FCSS and CSS liquid shares a similar trend against T. The equations used to obtain the FCSS and CSS plots are Eqs. (2) and (3) respectively.

FIG. 3: The Hall-Lorenz ratio in *ab*-planes, L_H^{ab} gives a convincing T-linear effect solely as a result of charge and spin separation. Exponential deviation from this linearity could arise from either E_I and/or E_F depending whether $E_I > E_F > T_c$ or $T_c < E_I < E_F$. The theoretical plots for FCSS and CSS liquid, neglecting spinon-pairing are given in a), b) and c), d) respectively. Equation (5) for FCSS and Eq. (6) for pure CSS liquid have been employed.

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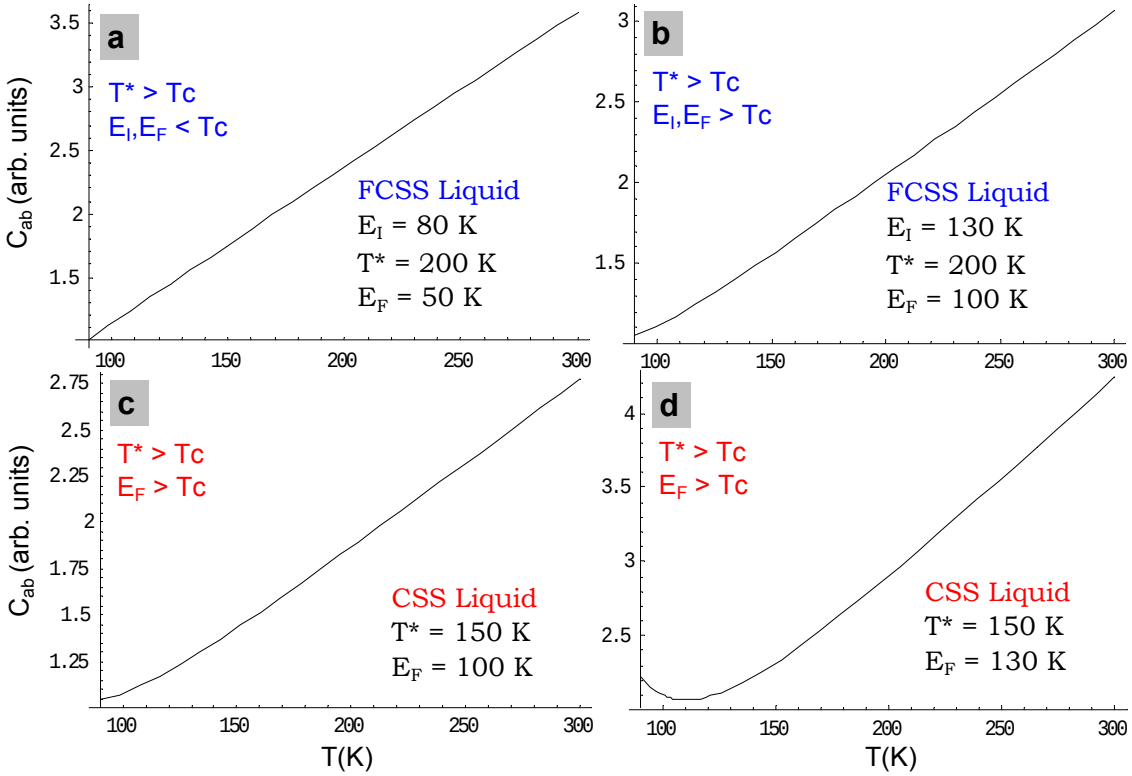
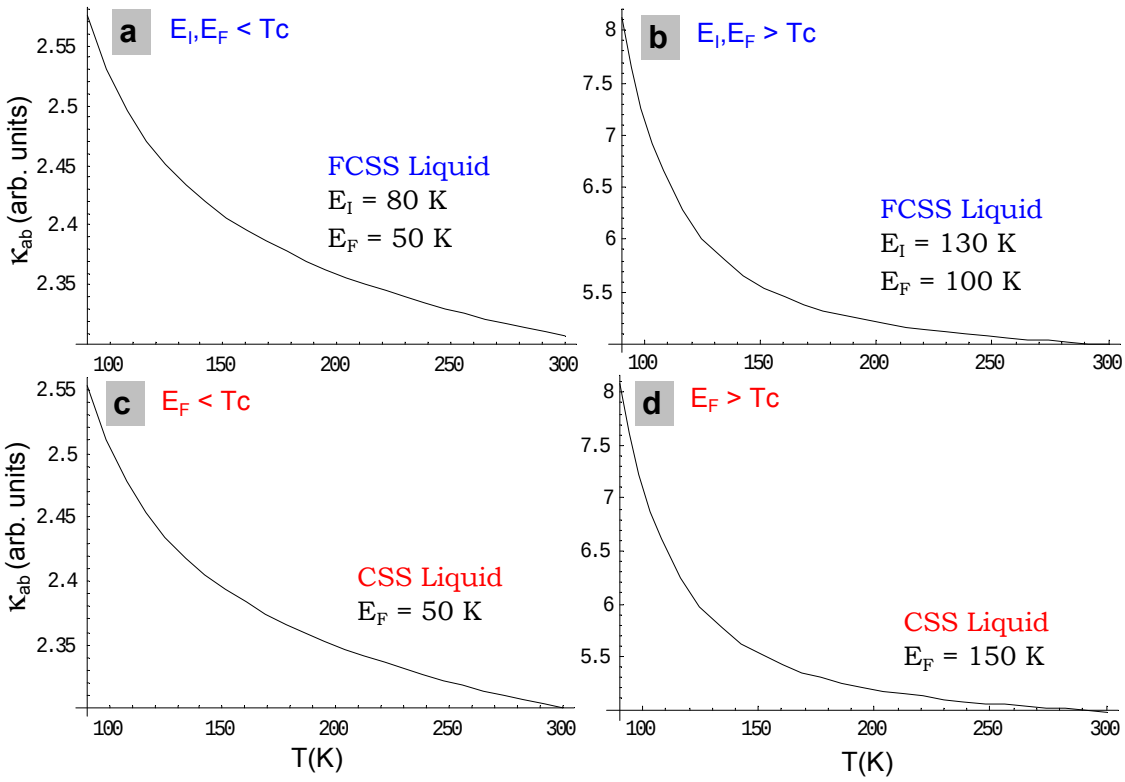
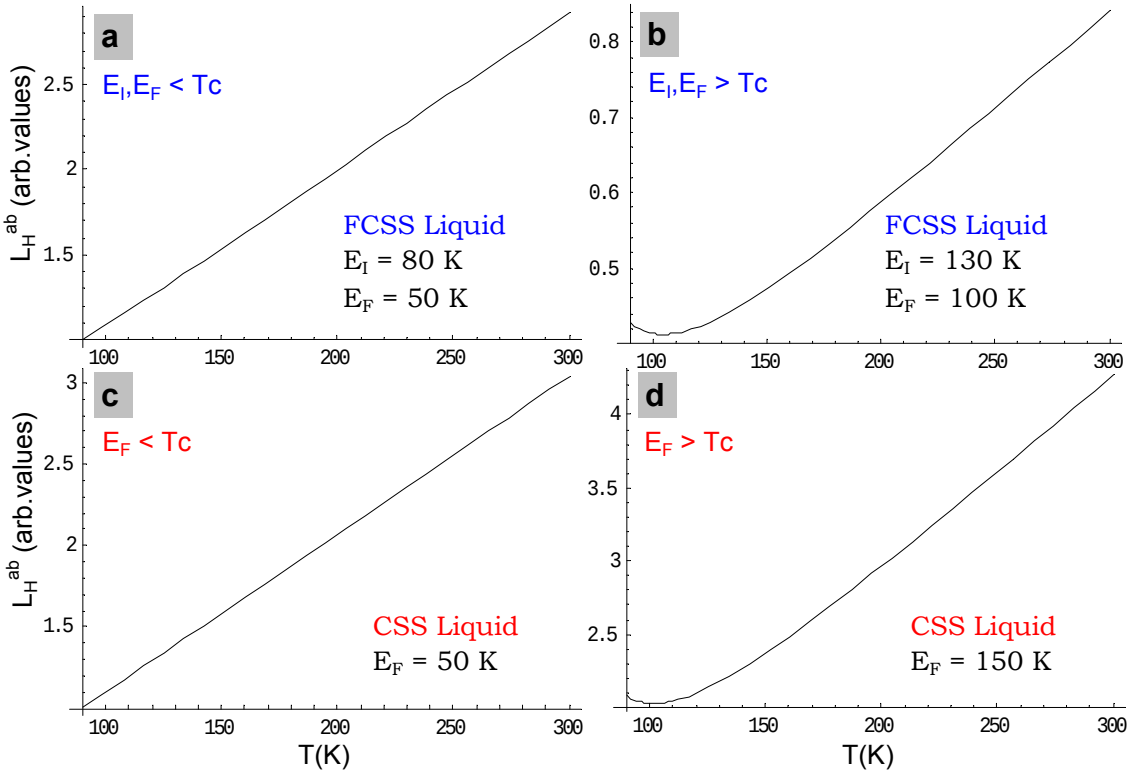

a
T* > Tc
E_I,E_F < Tc
FCSS Liquid
E_I = 80 K
T* = 200 K
E_F = 50 K
b
T* > Tc
E_I,E_F > Tc
FCSS Liquid
E_I = 130 K
T* = 200 K
E_F = 100 K
c
T* > Tc
E_F > Tc
CSS Liquid
T* = 150 K
E_F = 100 K
d
T* > Tc
E_F > Tc
CSS Liquid
T* = 150 K
E_F = 130 K
C_ab (arb. units)
T(K)


a
E_I,E_F < Tc
FCSS Liquid
E_I = 80 K
E_F = 50 K
b
E_I,E_F > Tc
FCSS Liquid
E_I = 130 K
E_F = 100 K
c
E_F < Tc
CSS Liquid
E_F = 50 K
d
E_F > Tc
CSS Liquid
E_F = 150 K
κ_ab (arb. units)
T(K)


a
$E_I, E_F < Tc$
FCSS Liquid
E_I = 80 K
E_F = 50 K
b
$E_I, E_F > Tc$
FCSS Liquid
E_I = 130 K
E_F = 100 K
c
$E_F < Tc$
CSS Liquid
E_F = 50 K
d
$E_F > Tc$
CSS Liquid
E_F = 150 K
L_H^{ab} (arb.values)
T(K)